IE-231 In-Class Activity - Week 12 - Solutions

Dec 12, 2017

1. Two friends (A and B) agree to meet on 4:00 PM. A usually arrives between 5 minutes early and 5 minutes late. B usually arrives between 5 minutes early and 15 minutes late. Their times of arrival are independent from each other.

    a) What is the probability that B arrives definitely later than A?
    b) What is the expected time that A waits B?
    c) What is the probability that both meet early?

    a) $P(B > 5) = \dfrac{5 - (-5)}{15 - (-5)} = 1/2$

    b) $E[B] - E[A] = 5 - 0 = 5$

    c) $P(B < 0, A < 0) = P(B < 0)P(A < 0) = 5/20 * 5/10 = 1/8$

2. There are three computers, which provide answers to questions with speed according to exponential distribution with means ($1/\lambda$) 6, 4 and 3 per hour, respectively. What is the probability that at least one machine provides an answer within the first hour?

$$P(X < x) = 1 - e^{-\lambda x}$$

$$P(X > x) = e^{-\lambda x}$$

The solution is 1 - no machine provides an answer within the hour $1 - P(X_1 > 1, X_2 > 1, X_3 > 1)$.

$$1 - P(X_1 > 1, X_2 > 1, X_3 > 1) = 1 - e^{-\lambda_1 - \lambda_2 - \lambda_3}$$

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## [1] 2.260329e-06
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3. Time between customer arrivals in a cafe is exponential with the mean value of 6 minutes.

a) What is the probability that no customers arrive in 15 minutes?

$P(X > 15) = 1 - P(X < 15) = 1 - (1 - e^{-\lambda x}) = e^{-15/6} = 0.08$

b) What is the inter-arrival time if the probability of a customer to arrive is 0.9?

$P(X > t) = e^{-t/6} = 0.1$, then $t = 13.81$.

c) What is the probability that 10 customers arrive in the first hour?

$\lambda * t = 1/6 * 60 = 10$

$P(X = 10) = \dfrac{e^{-\lambda * t}(\lambda t)^1 0}{10!} = 0.125.$

d) What is the probability of getting the first customer in 15 minutes if no customer arrived in the first 10 minutes?

$P(X < 15|X > 10) = 1 - P(X > 15|X > 10) = 1 - P(X > 5) = 1 - e^{-5/6} = 0.565$ (memoryless property)

Hint: Check the relationship between Poisson and Exponential distributions.

4. A pack of flour contains 1 kg of flour. Though a flour pouring machine has a standard deviation of 50 gr.

a) What is the probability that a randomly selected package contains between 925-1075 grams of flour?
b) If a proper flour package should contain between 1000-x and 1000+x grams of flour, what should x be that 80% of the packages are deemed proper?
c) Your customer strictly declared that 95% of the packages should contain at least 1000 grams of flour, so you should adjust the mean value. What should be the new mean value?

We have $\mu = 1000,\ \sigma = 50$. Define $\Phi(.)$ as the cdf of the standard normal distribution.

a) $\Phi(\dfrac{1075 - 1000}{50}) - \Phi(\dfrac{925 - 1000}{50})$
b) Find an x that $\Phi(\dfrac{1075 - 1000}{50}) - \Phi(\dfrac{925 - 1000}{50}) = 0.8$ approximately. The answer is more like how many standard deviations. We can find it by searching for a quantile of $\alpha = (1-0.8)/2 = 0.1$. Then find the y $\Phi(y) = 1 - \alpha$. $y = 1.282$ so reverse the procedure from standard normal to $\mathrm{N}(\mu = 1000,\ \sigma = 50)$ by $y * \sigma$ to find x.
c) Your $\Phi(\dfrac{1000 - \mu}{50}) = 0.05$. The quantile value for 0.05 is -1.645. So, 1000 - 50*(-1.645) = 1082.25.

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## [1] 0.8663856
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## [1] 64.07758
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## [1] 1082.243
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5. There are two different roads to get to Sarıyer. Road A takes 35 minutes on average with standard deviation 5 minutes. Road B takes 32 minutes on average with standard deviation 8 minutes.

    a) Which road has the higher advantage if one wants to reach Sarıyer in 42 minutes?
    b) What is the maximum time of arrival with 90% probability? Calculate for each road.

a) For Road A it will take 42 minutes or less with probability $\Phi(\dfrac{42-35}{5}) = \Phi(1.4) = 0.92$. For Road B, $\Phi(\dfrac{42-32}{8}) = \Phi(1.4) = 0.89$. Take Road A. b) It is the quantile value of $\alpha = 0.9$. Then we need to find $\Phi^{-1}(\alpha) = 1.28 = \dfrac{x-\mu}{\sigma}$. For Road A $1.28 * 5 + 35 = 41.4$, B $1.28 * 8 + 32 = 42.2$.

Note: b is phrased weakly. It is also possible to understand it as a probability interval as the probability of an event occurring in normal distribution happens in intervals (i.e. originates from mu). In that case the quantile max value is $\Phi^{-1}(0.95) = 1.64$ as there should be 5% slack at each side of the distribution.