

# IE-231 In-Class Activity - Week 6

Oct 30, 2017

This is a graded in-class assignment with peer review. **One submission per group on paper.** Do a clean work, your style will be evaluated too. Take a snapshot of your work after peer review. Check the details of peer review guidelines on Bilgi Learn.

## Question 1

A boutique shop offers three types of breads; olive, rye and white. 15% of its customers buy olive bread, 55% rye bread and the rest white bread. What is the probability that the first 10 customers of the shop buys 2 olive, 5 rye and 3 white bread?

*Solution:* Multinomial distribution.

$$\binom{10}{2, 5, 3} 0.15^2 * 0.55^5 * 0.3^3 = 0.0770478$$

## [1] 0.0770478

## Question 2

An urn contains 32 balls; 18 red, 14 white. If I take 8 balls randomly from the urn, what is the probability of getting 4 red balls?

*Solution:* Hypergeometric distribution.

$$\frac{\binom{18}{4} \binom{14}{4}}{\binom{32}{8}} = 0.2912125$$

## [1] 0.2912125

## Question 3

A UNICEF activist asks bypassers to contribute to their efforts. Each of her attempts has a 10% chance of success. What is the probability that she got the first contribution at the 10th attempt?

*Solution:* Geometric distribution.

$$(0.9)^9 * (0.1) = 0.03874205$$

## [1] 0.03874205

## Question 4

Suppose there are 10 cards in a deck numbered from 1 to 10. If I draw four cards out of the deck without putting them back, what is the probability that they are in increasing order (e.g. 3-4-5-8)?

*Solution:* Increasing order is a special order. There is only one decreasing order in every permutation set. For instance

$$\begin{aligned} &8 - 5 - 4 - 3 \\ &8 - 5 - 3 - 4 \\ &8 - 4 - 5 - 3 \\ &4 - 5 - 3 - 8 \\ &\dots \\ &*\ * 3 - 4 - 5 - 8 * * \end{aligned}$$

So, the answer is  $1/(4!) = 1/24$ .

## Question 5

A tennis player has the probability of 0.15 to ace a serve (i.e. he serves and gets the point without the opponent touching the ball) at each shot. His trainer challenged him that he will pay the square of the aces he makes out of 10 serves (e.g. if he serves 8 aces out of 10, the trainer pays him  $8^2 = 64TL$ ). But, if he serves less than or equal to 6 aces, the player should pay the trainer 5 TL for each ace he missed (e.g. 4 aces 6 misses, player pays up  $6*5 = 30TL$ ).

- What is the maximum amount of money he can get?
- What is the expected earnings of the player?
- What is the variance of his earnings?

*Solution:*

a) 10 out of 10 means  $10^2 = 100$ .

b)  $g(X = x) = x^2$  if  $x \geq 7$ ,  $f(g(X) = x^2) = f(X = x) = \binom{10}{x} 0.15^x * 0.85^{(10 - k)}$ , so  $E[g(x)] = \sum_{i=0}^1 0g(x)f(x)$ .

$g(X = x) = -5 * x$  if  $x \leq 6$ .

So  $49 * \binom{10}{7} 0.15^7 * 0.85^3 + 64 * \binom{10}{8} 0.15^8 * 0.85^2 + 81 * \binom{10}{9} 0.15^9 * 0.85 + 100 * (0.15)^{10} - 20 * \binom{10}{6} 0.15^6 * 0.85^4 - 25 * \binom{10}{5} 0.15^5 * 0.85^5 - 30 * \binom{10}{4} 0.15^4 * 0.85^6 - 35 * \binom{10}{3} 0.15^3 * 0.85^7 - 40 * \binom{10}{2} 0.15^2 * 0.85^8 - 45 * \binom{10}{1} 0.15^1 * 0.85^9 - 50 * 0.85^{10} = -42.5TL$ .

## [1] -42.4913

c)  $V(X) = E[X^2] - (E[X])^2$ . We know that  $E[X] = -42.5$ , then  $(E[X])^2 = 1805.51$ . Also  $E[X^2] = 1838.434$  and  $V(X) = 32.92$ .

## [1] 32.92423