IE-231 In-Class Activity - Week 10 - Solutions

Due Date Apr 18, 2017, 14:00

This is a graded in-class assignment. Show all your work in R Markdown files. Submit compiled Word files only.

1. Patients arrive at the doctor's office according to Poisson distribution with $\lambda = 2/\text{hour}$.

   a) What is the probability of getting less than or equal to 2 patients within 2 hours?
   b) Suppose each arriving patient has 50% chance to bring a person to accompany. There are 10 seats in the waiting room. At least many hours should pass that there is at least 50% probability that the waiting room is filled with patients and their relatives?

Solution

a) $P(X \leq 2|\lambda t = 2) = \sum_{i=0}^{2} \dfrac{e^{-\lambda t}(\lambda t)^i}{i!}$

```
#cdf of poisson
ppois(2,lambda=2*2)
```

```
## [1] 0.2381033
```

b) First let's define the problem. Define n_p as the number of patients and n_c is the number of company. We want $n_p + n_c \geq 10$ with probability 50% or higher for a given t^*. Or to paraphrase, we want $n_p + n_c \leq 9$ w.p. 50% or lower.

   What is n_c affected by? n_p. It is actually a binomial distribution problem. $P(n_c = i|n_p) = \binom{n_p}{i}(0.5)^i * (0.5)^{n_p - i}$. It is even better if we use cdf $P(n_c \leq k|n_p) = \sum_{i=0}^{k} \binom{n_p}{i}(0.5)^i * (0.5)^{n_p-i}$.

   We know the arrival of the patients is distributed with poisson. So, $P(n_p = j|\lambda t^*) = \dfrac{e^{-\lambda t}(\lambda t)^j}{j!}$. So $P(j + k \leq N) = \sum_{a=0}^{j} P(n_p = a|\lambda t^*) * P(n_c \leq N - a|n_p = a)$. Remember it is always $n_c \leq n_p$.

```
#Let's define a function
calculate_probability<-function(N=9,t_star=1,lambda=2){
    #N is the max desired number of patients
    the_prob<-0
    for(n_p in 0:N){
        the_prob <- the_prob + dpois(n_p,lambda=lambda*t_star)*pbinom(q=min(N-n_p,n_p),size=n_p,prob
    }

    return(the_prob)

}

#Try different t_stars so probability is below 0.5
calculate_probability(t_star=2)
```

```
## [1] 0.8631867
```

```
calculate_probability(t_star=3)
```

```
## [1] 0.5810261
```

```
calculate_probability(t_star=3.3)
```

```
## [1] 0.4905249
```

2. Two friends (A and B) agree to meet on 4:00 PM. A usually arrives between 5 minutes early and 5 minutes late. B usually arrives between 5 minutes early and 15 minutes late. Their times of arrival are independent from each other.

   a) What is the probability that B arrives definitely later than A?
   b) What is the expected time that A waits B?
   c) What is the probability that both meet early?

Solution

a) $P(B > 5) = \dfrac{5 - (-5)}{15 - (-5)} = 1/2$
b) $E[B] - E[A] = 5 - 0 = 5$
c) $P(B < 0, A < 0) = P(B < 0)P(A < 0) = 5/20 * 5/10 = 1/8$

3. There are three computers, which provides answers to questions with speed according to exponential distribution with means ($1/\lambda$) 6, 4 and 3 per hour, respectively. What is the probability that at least one machine provides an answer within the first hour?

Solution

$$P(X < x) = 1 - e^{-\lambda x}$$

$$P(X > x) = e^{-\lambda x}$$

The solution is 1 - no machine provides an answer within the hour $1 - P(X_1 > 1, X_2 > 1, X_3 > 1)$.

$$1 - P(X_1 > 1, X_2 > 1, X_3 > 1) = 1 - e^{-\lambda_1 - \lambda_2 - \lambda_3}$$

```
1 - pexp(1,rate=3+4+6)
```

```
## [1] 2.260329e-06
```

4. A pack of flour contains 1 kg of flour. Though a flour pouring machine has a standard deviation of 50 gr.

   a) What is the probability that a randomly selected package contains between 925-1075 grams of flour?
   b) If a proper flour package should contain between 1000-x and 1000+x grams of flour, what should x be that 80% of the packages are deemed proper?
   c) Your customer strictly declared that 95% of the packages should contain at least 1000 grams of flour, so you should adjust the mean value. What should be the new mean value?

Solution

We have $\mu = 1000,\ \sigma = 50$. Define $\Phi(.)$ as the cdf of the standard normal distribution.

a) $\Phi(\dfrac{1075 - 1000}{50}) - \Phi(\dfrac{925 - 1000}{50})$
b) Find an x that $\Phi(\dfrac{1075 - 1000}{50}) - \Phi(\dfrac{925 - 1000}{50}) = 0.8$ approximately. The answer is more like how many standard deviations. We can find it by searching for a quantile of $\alpha = (1 - 0.8)/2 = 0.1$. Then find the y $\Phi(y) = 1 - \alpha$. $y = 1.282$ so reverse the procedure from standard normal to N($\mu = 1000,\ \sigma = 50$) by $y * \sigma$ to find x.
c) Your $\Phi(\dfrac{1000 - \mu}{50}) = 0.05$. The quantile value for 0.05 is -1.645. So, 1000 - 50*(-1.645) = 1082.25.

```
#a
pnorm(1075,mean=1000,sd=50)-pnorm(925,mean=1000,sd=50)
```

```
## [1] 0.8663856
```

```r
#b
qnorm(0.9,sd=50)
```

```
## [1] 64.07758
```

```r
#c
1000-qnorm(0.05,sd=50)
```

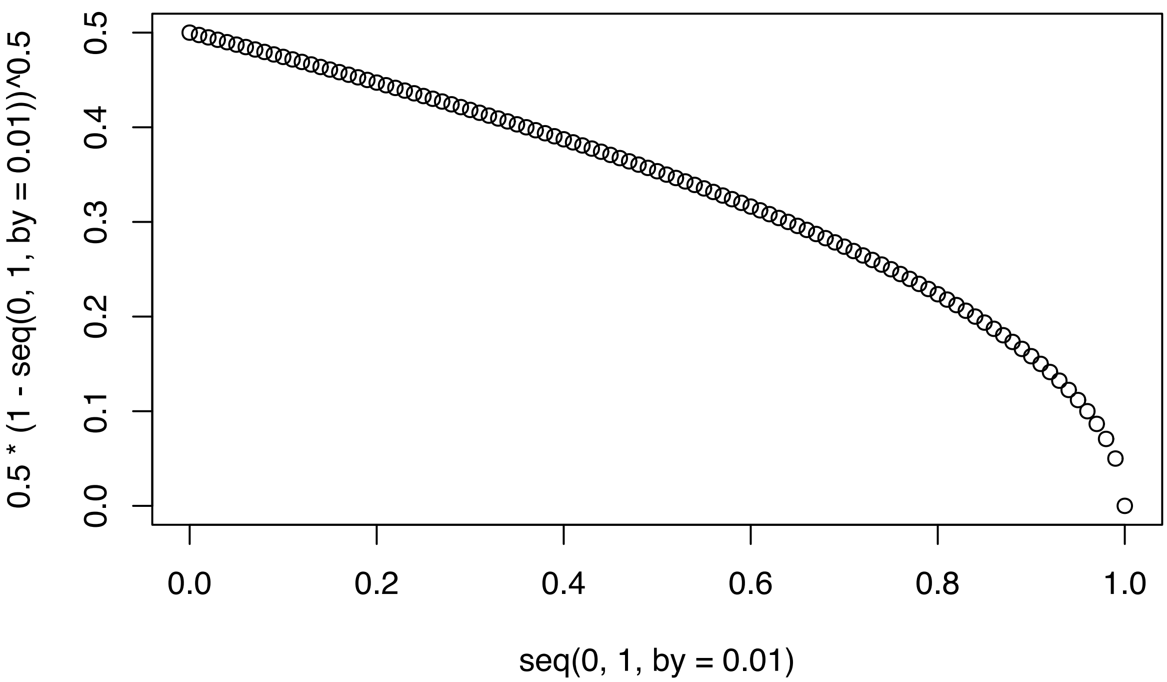
```
## [1] 1082.243
```

5. Suppose the the pdf of a random variable x is $f(x) = \frac{a}{(1-x)^0.5}$ for $0 < x < 1$ and 0 for other values of x.

a) Find the constant a and sketch pdf with R.

Integral of $f(x)$ should be 1. So $\int_0^1 f(x)dx = a*(-2(1-x)^{1/2})|_0^1 = 2a$. Then $a = 1/2$.

```r
plot(seq(0,1,by=0.01),0.5*(1-seq(0,1,by=0.01))^0.5)
```



b) Find cdf value of F(X < 1/4).

Use the same integral $\int_0^{1/4} f(x)dx = -(1-x)^1/2dx = 1-(3/4)^(1/2) = 0.134$