

# Quiz 01

IE-231

Mar 14, 2017

Name Surname:

Student Number:

You don't need to find exact results. Just do the operations. You have 30 mins.

1. In how many ways can you arrange the letters of "SERENDIPITY"?

- a) Any order?
- b) Vowels together?

Solution: SERENDIPITY has 11 letters: 4 vowels, 7 consonants. There are 2 Es and 2 Is.

a) Remember the same letters should be accounted for.  $\frac{11!}{2!2!}$

b) Suppose vowels are a single word (i.e. X is any permutation of EEII), call it a special word. So a permutation can be SRNDPTYX, another would be NRSXDPTY. Also "within" X, vowels can get different permutations (e.g. EIEI or IEEI). If we replace X with the vowels some proper permutations would be SRNDPTYEIEI, SRNDPTYIEEI, NRSEIEIDPTY, NRSIEEIDPTY.

First assume vowels do not change place. Then there are 8! permutations. Now calculate vowel permutations  $\frac{4!}{2!2!}$ . For each vowel permutation and a special word (i.e. SRNDPTYX) permutation, there is a valid permutation (i.e. vowels together).

So the answer is  $8! * \frac{4!}{2!2!}$ .

2. In a box there are 12 balls, 4 white 8 black. If I randomly pick 5 balls from the box, what is the probability that it will be 1 white and 4 blacks?

Remember Hypergeometric distribution.

$$\frac{\binom{4}{1}\binom{8}{4}}{\binom{12}{5}}$$

3. John's basketball shots score with probability 0.8. What is the probability that at least three scores among the first 10 shots?

Solution: It is a binomial distribution problem. A single probability (i.e. scoring exactly k shots) can be calculated as follows  $\binom{n}{k}p^k(1-p)^{n-k}$ . There are two ways to calculate the correct answer. First one is to calculate scores from 3 to 10, second one is calculate from 0 to 2 and subtract from total probability (i.e. 1). Below expression belongs to the second solution.

$$1 - P(X = 0) - P(X = 1) - P(X = 2) = 1 - (0.2)^{10} - 10 * (0.8) * (0.2)^9 - \binom{10}{2} * (0.8)^2 * (0.2)^8$$